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Anomaly cancellation in supergravity with Fayet–Iliopoulos couplings

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Abstract: We review and clarify the cancellation conditions for gauge anomalies which occur when $\mathcal{N} = 1,\ D = 4$ supergravity is coupled to a Kähler non-linear sigma-model with gauged isometries and Fayet–Iliopoulos couplings. For a flat sigma-model target space and vanishing Fayet–Iliopoulos couplings, consistency requires just the conventional anomaly cancellation conditions. A consistent model with non-vanishing Fayet–Iliopoulos couplings is unlikely unless the Green–Schwarz mechanism is used. In this case the U(1) gauge boson becomes massive and the D-term potential receives corrections. A Green–Schwarz mechanism can remove both the abelian and certain non-abelian anomalies in models with a gauge non-invariant Kähler potential.

Keywords: Supergravity Models, Anomalies in Field and String Theories.

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Contents

1. Introduction

In $d = 4$, $\mathcal{N} = 1$ supergravity fermions couple to the Kähler connection and the sigma-model connection. These are composite connections constructed from elementary scalars and gauge potentials. Since the gauge transformations are embedded in the geometry of a sigma-model manifold these connections are in general not gauge invariant. In particular the Kähler connection acts as an additional abelian gauge field that effectively gauges the $\mathrm{U}(1)_R$ symmetry. Fermions have chiral couplings to both elementary and composite connections, so there may be anomalies which threaten the consistency of the theory. This subject has been investigated both in supergravity models, see [1 − 3] for early work, and string compactifications [4 − 6].

A general analysis of the quantum consistency conditions for supergravity was recently presented in [7], in which effects of the composite connections were emphasized. It follows from the gauge field equations of motion $D_\mu F^{a\mu\nu} = J^{a\nu}$ that

$$0 \equiv D_\nu D_\mu F^{a\mu\nu} = D_\nu J^{a\nu}\,. \tag{1.1}$$

The left side vanishes identically, so the current $J^{a\nu}$ must be conserved. Conservation holds in the classical theory, but can be violated in the quantum theory by anomalies, viz. $D_\nu J^{a\nu} = \mathcal{A}^a \neq 0$. The quantum theory is inconsistent unless the anomalies are cancelled. The detailed consistency conditions for supergravity were found in [7] and expressed in terms of covariant anomalies.

The purpose of this paper is to unravel the structure of the consistency conditions and clarify the anomaly cancellation conditions required by quantum consistency of the theory. We focus here on theories with flat target spaces. We recast the consistency conditions in terms of consistent anomalies and consider the effects of finite local counter terms. This reduces the results of [7] to a set of physically necessary consistency conditions. We further discuss the Green–Schwarz mechanism, which requires additional degrees of freedom.

Two distinct consistency conditions, abelian and non-abelian, arise from (1.1) depending on whether the current $J^{a\nu}$ is abelian or non-abelian. We are especially concerned with *mixed anomalies*. Using $C_{\mu\nu}$ and $F^a_{\mu\nu}$ for the abelian and non-abelian field strengths, examples of (covariant) mixed anomalies are terms involving the non-abelian fields like $\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\, F_{\mu\nu}F_{\rho\sigma}$ in the divergence of the abelian current ("mixed abelian anomalies"), or $\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\, T^a C_{\mu\nu}F_{\rho\sigma}$ in the divergence of the non-abelian current ("mixed non-abelian anomalies"). The field strength $K_{\mu\nu} = \partial_\mu K_\nu - \partial_\nu K_\mu$ of the Kähler connection K_μ also appears in the mixed anomalies.

For supergravity theories with a flat sigma-model target space and a gauge group $G\times \mathrm{U}(1)$ we clarify the anomaly cancellation conditions, with and without Fayet–Iliopoulos couplings. The results are summarized and discussed at the end of section 3. In case of vanishing Fayet–Iliopoulos couplings, standard conditions on the matter content suffice to ensure consistency of the theory. But for general non-vanishing Fayet–Iliopoulos couplings consistency requires the Green–Schwarz mechanism.

The standard Green–Schwarz mechanism can only remove anomalies in abelian conservation laws.[1] Consequently, to ensure consistency, all mixed non-abelian anomalies must be cancelled by other means; either by finite local counter terms or by imposing conditions on the matter content of the theory. Furthermore, the Green–Schwarz mechanism requires that the abelian anomaly removed is gauge covariant. However, not all consistent mixed abelian anomalies are covariant. Local counter terms are needed to restructure them before the Green–Schwarz method is applied. We construct here finite local counter terms which have both required properties: (1) they remove completely the consistent mixed non-abelian anomalies, and (2) they simultaneously convert the consistent mixed abelian anomalies to covariant form. The final resulting consistency conditions are summarized in section 4, where we also comment on the effect of gravitational anomalies.

We focus in most of this work on Kähler potentials that are invariant under non-abelian gauge transformations. A simple model with a non-invariant Kähler potential is studied in section 5.1. Since the Kähler connection transform as a $\mathrm{U}(1)$ connection under non-abelian gauge transformations, a Green–Schwarz mechanism can be used to cancel certain anomalies in the non-abelian current conservation law.

This study is relevant to various models of cosmology and particle physics that make use of Fayet–Iliopoulos couplings, such as D-term inflation [8, 9], or supersymmetry breaking

[1]An exception to this is studied in section 5.1.

JHEP11(2006)068

via an anomalous U(1) [10], or the string solutions for so-called D-strings in [11 − 14], see also [15, 16]. In string theory Fayet–Iliopoulos couplings were discussed by Dine, Seiberg and Witten [17]. More recently, D-terms and Fayet–Iliopoulos couplings have played an important role in the context of moduli stabilization in string compactifications, see [18], following the proposal of Kachru, Kallosh, Linde, and Trivedi [19]. Our conclusion about the validity of such models at the quantum level is that Fayet–Iliopoulos couplings can only be consistent if one incorporates a Green–Schwarz mechanism to cancel residual anomalies. Such a Green–Schwarz mechanism modifies the physics by generating a mass for the gauge boson and by a contribution to the D-term potential.

The paper is organized as follows. In section 2 we review the covariant and consistent anomalies, and we show how finite local counter terms can be used to restructure mixed anomalies. We review in section 3 the consistency conditions derived in ref. [7], write the anomaly cancellation conditions in terms of the consistent anomaly, and include local counter terms to reduce the anomaly cancellation conditions. The Green–Schwarz mechanism is discussed in section 4. We discuss briefly in section 5 generalizations to non-gauge invariant Kähler potentials as well as the supersymmetrization of anomalies and local counter terms. We conclude in section 6 with a summary and discussion of our results.

2. Covariant vs. consistent anomalies

To derive the physically relevant anomaly cancellation conditions from the requirement of current conservation it is crucial to use the proper form of the anomalies and include all possible (finite) local counter terms into the Lagrangian. To provide the background material needed later we have included a short review section on consistent and covariant anomalies.

Consider the kinetic Lagrangian of a Weyl fermion, written as the left-handed component of a Dirac fermion, minimally coupled to a background gauge field V_μ,[2]

$$\mathcal{L} \;=\; \bar{\psi}\gamma^\mu D_\mu L\psi\,, \quad D_\mu L\psi \;=\; (\partial_\mu + V_\mu)L\psi\,, \tag{2.1}$$

where $L = \frac{1}{2}(1-\gamma_5)$, $R = \frac{1}{2}(1+\gamma_5)$ and $V_\mu = V_\mu^a T^a$. The T^a are anti-hermitian generators of a Lie algebra for a gauge group $\mathcal{G}$, which may contain U(1) factors. The action with (2.1) is invariant under chiral gauge transformations,

$$L\psi \to e^{-\theta(x)} L\psi\,, \qquad V_\mu \to e^{-\theta(x)} V_\mu e^{\theta(x)} + e^{-\theta(x)}\partial_\mu e^{\theta(x)} \tag{2.2}$$

with $\theta = \theta^a T^a$. Correspondingly, the left-chiral current is classically conserved,

$$j_\mu^a = -\bar{\psi}\gamma_\mu T^a L\psi\,, \quad 0 = D_\mu j^\mu = \partial_\mu j^\mu + [V_\mu, j^\mu]\,. \tag{2.3}$$

Classical symmetries and conservation laws receive corrections in the quantum theory due to anomalies.

[2]We concentrate on the left-chiral part of the gauge transformations. Generalizations are straight forward.

JHEP11(2006)068

2.1 Anomalies

The covariant (left-)chiral anomaly is

$$
\begin{aligned}
(D_\mu j^\mu)^a &= \frac{i}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[T^a V_{\mu\nu}V_{\rho\sigma}\Big] \\
&= \frac{i}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[T^a\partial_\mu\Big(V_\nu\partial_\rho V_\sigma + \frac{2}{3}V_\nu V_\rho V_\sigma\Big)\Big]\,,
\end{aligned}
\tag{2.4}
$$

with $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu]$. An anomaly reflects the gauge non-invariance of the effective action

$$
e^{-W[V_\mu]} = \Gamma[V_\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S[V_\mu,\bar{\psi},\psi]}\,,
\tag{2.5}
$$

where V_μ denotes a background gauge field. Defining the current $j^{a\,\mu} = -\delta\mathcal{L}[V_\mu]/\delta V^a_\mu$ one has

$$
\delta_\theta W[V_\mu] = \int d^4x\ \theta^a \mathcal{A}^a\,,
\tag{2.6}
$$

where $\mathcal{A}^a = \langle (D_\mu j^\mu)^a\rangle$ is the anomaly under the gauge transformation

$$
\delta_\theta V_\mu = D_\mu\theta = \partial_\mu\theta + [V_\mu, \theta]\ .
\tag{2.7}
$$

The Wess-Zumino consistency condition $[\delta_{\theta_1}, \delta_{\theta_2}]W[V_\mu] = \delta_{[\theta_1,\theta_2]}W[V_\mu]$ requires

$$
\delta_{\theta_1}(\theta^a_2\mathcal{A}^a) - \delta_{\theta_2}(\theta^a_1\mathcal{A}^a) = [\theta_1, \theta_2]^a\mathcal{A}^a
\tag{2.8}
$$

for the anomaly [20]. It is not satisfied for a simple non-abelian gauge group by the covariant anomaly (2.4), since a factor of 2 appears on the right side of eq. (2.8), see e.g. [21].

The form of the anomaly that does satisfy (2.8) is known as the *consistent anomaly* (Bardeen [22], Gross and Jackiw [23]). The consistent anomaly follows from a Bose symmetric regularization of the triangle and quadrangle Feynman diagrams for correlation functions of the potentials V_μ. For the left-chiral current (2.3) it can be written[3]

$$
(D_\mu j^\mu)^a = \frac{i}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[T^a\,\partial_\mu\Big(V_\nu\,\partial_\rho V_\sigma + \frac{1}{2}V_\nu V_\rho V_\sigma\Big)\Big]\,.
\tag{2.9}
$$

Note that (2.4) does not transform covariantly. For abelian V_μ, the cubic term is absent and the anomalies (2.4) and (2.9) differ by an overall factor of $\frac{1}{3}$ which accounts for Bose symmetry of the triangle amplitude.

The reason Wess-Zumino consistency fails for the covariant anomaly is that the current on the left-hand side of (2.4) is not the variation of the effective action. Bardeen and Zumino showed [24] that the current in (2.4) differs from the consistent current by a polynomial local in the gauge potential. For a simple non-abelian gauge group the Bardeen-Zumino polynomial cannot be written as the gauge variation of a finite local counter term added to the effective action. Hence the covariant and the consistent anomalies are not physically equivalent.[4] For the purpose of analyzing the consistency conditions [7] for currents which are sources of gauge fields, the relevant form of the anomaly is the consistent anomaly (2.9).

[3]For a right-chiral current the overall sign of the anomaly changes.

[4]Nonetheless, the vanishing of the covariant and the consistent anomalies for a simple gauge group requires the same condition, namely $\mathrm{tr}\,T^a\{T^b, T^c\} = 0$.

JHEP11(2006)068

2.2 Local counter terms

Having established that the two forms, (2.4) and (2.9), of anomalies are not equivalent for a simple non-abelian gauge group, we point out how one can interpolate between them for a mixed anomaly.[5] This will later be crucial for applying the Green–Schwarz mechanism.

Consider a gauge group which is the product of a single $\mathrm{U}(1)$ factor and a simple non-abelian group G, $\mathcal{G} = G \times \mathrm{U}(1)$. Write the gauge field $V_\mu = A_\mu^a T^a + iQC_\mu$, where the T^a are anti-hermitian generators of G and Q is the charge under $\mathrm{U}(1)$. We use $F_{\mu\nu}^a$ for the non-abelian field strength and $C_{\mu\nu}$ for the abelian field strength. Inserting $V_\mu = A_\mu^a T^a + iQC_\mu$ into the expression for the consistent anomaly (2.9) and covariant anomaly (2.4), we pick up terms which are purely abelian or purely non-abelian anomalies as well as mixed anomalies. We write this

$$\begin{aligned}(D_\mu j^\mu)^a &= \mathcal{A}^a = \mathcal{A}^a_{\text{non-ab}} + \mathcal{A}^a_{\text{mixed}}\,,\\ (D_\mu j^\mu)^Q &= \mathcal{A}^Q = \mathcal{A}^Q_{\text{abel}} + \mathcal{A}^Q_{\text{mixed}}\,,\end{aligned} \tag{2.10}$$

where $j_\mu^a = -\delta\mathcal{L}/\delta A_\mu^a = -i\bar{\psi}\gamma_\mu T^a L\psi$ and $j_\mu^Q = -\delta\mathcal{L}/\delta C_\mu = -i\bar{\psi}\gamma_\mu iQ\,L\psi$ are the non-abelian and abelian currents. Below subscripts "cov" or "con" indicate whether a given term in the anomalies is written in the covariant form (2.4) or the consistent form (2.9).

To be explicit, we list the mixed anomalies $\mathcal{A}^a_{\text{mixed}}$ and $\mathcal{A}^Q_{\text{mixed}}$ in covariant and consistent form respectively,

$$\begin{aligned}\mathcal{A}^Q_{\text{mixed cov}} &= \frac{i}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\, iQ\,F_{\mu\nu}F_{\rho\sigma} = \frac{i}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[iQ\partial_\mu\Big(A_\nu\,\partial_\rho A_\sigma + \frac{2}{3}A_\nu A_\rho A_\sigma\Big)\Big]\,,\\ \mathcal{A}^Q_{\text{mixed con}} &= \frac{i}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[iQ\partial_\mu\Big(A_\nu\,\partial_\rho A_\sigma + \frac{1}{2}A_\nu A_\rho A_\sigma\Big)\Big]\,,\\ \mathcal{A}^a_{\text{mixed cov}} &= \frac{i}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[T^a iQ\,C_{\mu\nu}F_{\rho\sigma}\Big]\,,\\ \mathcal{A}^a_{\text{mixed con}} &= \frac{i}{12\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[T^a iQ\partial_\mu\Big(C_\nu\,\partial_\rho A_\sigma + \frac{1}{4}C_\nu A_\rho A_\sigma\Big)\Big]\,.\end{aligned} \tag{2.11}$$

There are two candidate polynomials in C_μ and A_μ from which finite local counter terms in the Lagrangian can be constructed,

$$\begin{aligned}\mathcal{L}_1 &= -\frac{i}{12\pi^2}\,\epsilon^{\mu\nu\rho\sigma}C_\mu\mathrm{tr}\Big[iQA_\nu\partial_\rho A_\sigma\Big]\,,\\ \mathcal{L}_2 &= -\frac{i}{12\pi^2}\,\epsilon^{\mu\nu\rho\sigma}C_\mu\mathrm{tr}\Big[iQA_\nu A_\rho A_\sigma\Big]\,.\end{aligned} \tag{2.12}$$

Their gauge variations under non-abelian gauge transformations (2.7) are (up to total derivatives)

$$\begin{aligned}\delta_\theta\mathcal{L}_1 &= -\frac{i}{12\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[iQ\theta\Big(\partial_\mu C_\nu\,\partial_\rho A_\sigma - 2\partial_\mu(C_\nu A_\rho A_\sigma)\Big)\Big]\,,\\ \delta_\theta\mathcal{L}_2 &= -\frac{i}{4\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[iQ\theta\partial_\mu(C_\nu A_\rho A_\sigma)\Big]\,.\end{aligned} \tag{2.13}$$

[5]Recent interesting work [25] studies local counter terms and the Green–Schwarz mechanism in connection to anomalous $\mathrm{U}(1)$'s.

One may add these counter terms with arbitrary coefficients to the Lagrangian. This would modify the non-abelian current conservation law by terms proportional to (2.13). The unique combination

$$\mathcal{L}_{\rm ct} \;=\; \mathcal{L}_1 + \frac{3}{4}\mathcal{L}_2 \tag{2.14}$$

precisely cancels the non-abelian mixed anomaly in the consistent form,

$$\delta_\theta \mathcal{L}_{\rm ct} \;=\; -\,\theta^a \mathcal{A}^a_{\text{mixed con}}\,. \tag{2.15}$$

Under abelian gauge variations

$$\delta_\Lambda C_\mu = \partial_\mu \Lambda \tag{2.16}$$

the counter term gives

$$\delta_\Lambda \mathcal{L}_{\rm ct} = -\Lambda(\mathcal{A}^Q_{\text{mixed con}} - \mathcal{A}^Q_{\text{mixed cov}})\,, \tag{2.17}$$

i.e. it “rotates” the consistent form of the abelian mixed anomaly into covariant form. As discussed in the Introduction, this is essential for the Green–Schwarz mechanism.

The gauge variations (2.13) of the counter terms yield total derivatives, but the covariant mixed anomaly $\mathcal{A}^a_{\text{mixed cov}}$ given in (2.11) involves $\epsilon^{\mu\nu\rho\sigma}{\rm tr}\, T^a C_{\mu\nu} F_{\rho\sigma}$ which is not a total derivative. Hence the non-abelian variations of the counter terms (2.13) could never fully cancel $\mathcal{A}^a_{\text{mixed cov}}$, it is therefore crucial to use $\mathcal{A}^a_{\text{mixed con}}$.

3. Kähler anomalies in supergravity

We start with a summary of some relevant structures of the supergravity Lagrangian and of results of [7].

3.1 Supergravity and composite connections

We consider theories with a supergravity multiplet (e^i_μ, Ψ_μ) coupled to gauge multiplets (V^a_μ, λ^a) and chiral multiplets $(z^\alpha, L\psi^\alpha)$. We write the gravitino Ψ_μ and the gauginos λ^a as four-component Majorana spinors, and the Weyl spinors of the chiral multiplets are written with projectors L, R. We write the action $S = \int d^4x\, \sqrt{-g}\, \mathcal{L}$ and use $\epsilon^{0123} = (-g)^{-1/2}$. More details, including the Lagrangian, are given in [7].

The scalar fields are complex coordinates on a Kähler manifold with Kähler potential $K = K(z, \bar{z})$ and metric $G_{\alpha\bar{\beta}} = K_{,\alpha\bar{\beta}}$ (a comma indicating a partial, a semi-colon a covariant derivative). In supergravity, isometries of the Kähler manifold generated by holomorphic Killing vectors, $X^{a\alpha}(z)$, $X^{a\bar{\alpha}}(\bar{z})$, can be gauged. Holomorphic Killing vectors can be expressed as gradients of a real Killing prepotential $D^a(z,\bar{z})$ as $D^a_{,\bar{\beta}} = iX^{a\alpha}G_{\alpha\bar{\beta}}$. For non-abelian gauge groups the prepotentials are uniquely determined by the requirement that they transform in the adjoint representation. For abelian gauge groups, there is an additional freedom of adding a constant, $D^a \to D^a + \xi^a$. These constants are the Fayet–Iliopoulos couplings of the theory.

The Kähler metric must be invariant under the isometry, and this requirement is exactly the Killing equation

$$\delta^a G_{\alpha\bar\beta} = X^a_{\alpha;\bar\beta} + X^a_{\bar\beta;\alpha} = 0\,. \tag{3.1}$$

However, the Kähler potential need not be invariant and transforms as

$$\delta^a K(z,\bar z) = X^{a\alpha}K_{,\alpha} + X^{a\bar\alpha}K_{,\bar\alpha} = F^a(z) + \bar F^a(\bar z)\;. \tag{3.2}$$

The holomorphic function $F^a(z)$ is related to D^a,

$$F^a = X^{a\alpha}K_{,\alpha} + iD^a\,. \tag{3.3}$$

The supergravity model contains an elementary gauge field V^a_μ for each isometry. They appear in covariant derivatives of the scalars as $D_\mu z^\alpha = \partial_\mu z^\alpha - V^a_\mu X^{a\alpha}$ and in the composite Kähler connection

$$K_\mu = \frac{1}{2i}\Big(K_{,\alpha}\, D_\mu z^\alpha + F^a V^a_\mu - \text{ c.c. }\Big) = \frac{1}{2i}\Big(K_{,\alpha}\,\partial_\mu z^\alpha - \text{ c.c. }\Big) + V^a_\mu D^a\;. \tag{3.4}$$

Under gauge transformations (2.7) for V^a_μ and $\delta z^\alpha = \theta^a(x)X^{a\alpha}$ for z^α, the Kähler connection transforms as a U(1) connection

$$\delta K_\mu = \partial_\mu(\theta^a \mathrm{Im}\, F^a(z))\;. \tag{3.5}$$

The non-invariance of K and the coupling of K_μ in the covariant derivatives of all fermions are the essential complicating factors of the anomaly analysis in supergravity.

The fermion covariant derivatives are

$$\begin{aligned}
D_\mu\Psi_\nu &= \Big(\nabla_\mu + \frac{1}{2}iK_\mu\gamma_5\Big)\Psi_\nu \;=\; \Big(\partial_\mu + \frac{1}{4}\omega_{\mu ij}\gamma^{ij} + \frac{1}{2}iK_\mu\gamma_5\Big)\Psi_\nu\,,\\
D_\mu\lambda^a &= \Big(\nabla_\mu\delta^{ac} + \frac{1}{2}iK_\mu\gamma_5\delta^{ac} + f^{abc}V^b_\mu\Big)\lambda^c\,,\\
D_\mu L\psi^\alpha &= \Big(\nabla_\mu\delta^\alpha_\beta + \Sigma^\alpha_{\beta\mu} + \frac{1}{2}iK_\mu\delta^\alpha_\beta - X^{a\alpha}{}_{,\beta}\,V^a_\mu\Big)L\psi^\beta\;.
\end{aligned} \tag{3.6}$$

The first line of (3.6) defines the derivative ∇_μ which includes the spin-connection. The other composite connection is the sigma-model connection $\Sigma^\alpha_{\mu\beta} = \Gamma^\alpha_{\beta\gamma}D_\mu z^\gamma$, where $\Gamma^\alpha_{\beta\gamma} = G^{\alpha\bar\delta}G_{\beta\bar\delta,\gamma}$ are the Kähler Christoffel connections. It will not be important for us. Note that $\frac{1}{2}K_\mu$ gauges a $\mathrm{U}(1)_R$ symmetry under which $L\Psi_\mu$ and $L\lambda^a$ have charge +1 and $L\psi^\alpha$ has charge −1. The gravitational coupling κ has been set to $\kappa = 1$ for simplicity, but it actually appears in the fermion covariant derivatives through $\kappa^2 K_\mu$.

The infinitesimal gauge transformations of the fermions are

$$\begin{aligned}
\delta\Psi_\mu &= -\frac{i}{2}\theta^a \mathrm{Im}\, F^a(z)\gamma_5\Psi_\mu\,,\\
\delta\lambda^a &= f^{abc}\lambda^b\theta^c - \frac{i}{2}\theta^b\mathrm{Im}\, F^b(z)\gamma_5\lambda^a\,,\\
\delta L\psi^\alpha &= \theta^a X^{a\alpha}{}_{,\beta}\,L\psi^\beta - \frac{i}{2}\theta^a \mathrm{Im}\, F^a(z)L\psi^\alpha\;.
\end{aligned} \tag{3.7}$$

The $\mathrm{Im}\, F^a$-terms compensate the transformation (3.5) of K_μ, while the other terms are standard transformations of gauginos and chiral fermions.

The chiral transformations (3.7) are anomalous and it is these anomalies which are studied in [7]. Consistency of the quantum theory requires that the following combination of anomalous current divergences must cancel:

$$0 = iY^a_{\alpha\bar\beta}\langle\nabla_\mu(\bar\psi^{\bar\beta}\gamma^\mu L\psi^\alpha)\rangle + \frac{1}{2}\langle\nabla_\mu(\bar\lambda^b f^{abc}\gamma^\mu\lambda^c)\rangle + \frac{1}{2}\mathrm{Im}\, F^a\langle\nabla_\mu N^\mu\rangle\,, \tag{3.8}$$

with

$$Y^a_{\alpha\bar\beta} = \frac{1}{2i}\Big(G_{\gamma\bar\beta}X^{a\gamma}{}_{,\alpha} - G_{\alpha\bar\gamma}X^{a\bar\gamma}{}_{,\bar\beta}\Big)\,. \tag{3.9}$$

The current N^μ is the $\mathrm{U}(1)_R$ current to which the Kähler connection couples, namely

$$N^\mu = -\frac{i}{2}\Big[2G_{\alpha\bar\beta}\bar\psi^{\bar\beta}\gamma^\mu L\psi^\alpha + \bar\lambda^a\gamma^\mu\gamma_5\lambda^a + \bar\Psi_\rho\gamma^{\rho\mu\nu}\gamma_5\Psi_\nu\Big]\,. \tag{3.10}$$

In (3.8) the brackets $\langle\ldots\rangle$ indicate the quantum anomalies of each current. These anomalies were computed in [7] as covariant anomalies using the Fujikawa method. The expressions for the anomalies are rather complicated in the general case, involving the field strengths of the full connections in (3.6). The consistency conditions (3.8) will be rewritten in terms of consistent anomalies in section 3.3.

3.2 Supergravity models with flat target space

In most of this paper we will restrict the treatment to models with flat target space and linearly realized gauge symmetries,

$$K(z,\bar z) = \delta_{\alpha\bar\beta}z^\alpha z^{\bar\beta}\,, \quad G_{\alpha\bar\beta} = \delta_{\alpha\bar\beta}\,, \quad X^{a\alpha}{}_{,\beta} = -T^{a\alpha}{}_\beta = -T^{ai}{}_j e_i^\alpha e_\beta^j\,. \tag{3.11}$$

The sigma-model connection then vanishes, $\Sigma^\alpha_{\beta\mu} = \Gamma^\alpha_{\beta\gamma}D_\mu z^\gamma = 0$. Although the Kähler potential is gauge invariant, $\mathrm{Im}F^a$ can be nonzero when there are Fayet–Iliopoulos couplings, i.e.

$$F^a = i\xi^a \tag{3.12}$$

for abelian factors of the gauge group. The Kähler connection (3.4) then becomes

$$K_\mu = \mathrm{Im}(\delta_{\alpha\bar\beta}z^{\bar\beta}D_\mu z^\alpha) + \xi^a V^a_\mu\,. \tag{3.13}$$

The T^a_{ij} are the anti-hermitian constant matrix generators of the gauge symmetry and we use

$$[T^a, T^b] = f^{abc}T^c\,, \quad f^{acd}f^{bcd} = \delta^{ab}C_2(G)\,, \quad \mathrm{tr}_r(T^aT^b) = -C(r)\delta^{ab}\,, \tag{3.14}$$

where "tr_r" indicates the trace over the irreducible representation r and "tr" the trace over the full spectrum of chiral fermions, not including the gauginos or the gravitino. In this limit (3.8) simplifies to

$$0 \;=\; -\langle\nabla_\mu(\bar\psi^i T^a_{ij}\gamma^\mu L\psi^j)\rangle + \frac{1}{2}\langle\nabla_\mu(\bar\lambda^b f^{abc}\gamma^\mu\lambda^c)\rangle + \frac{1}{2}\xi^a\langle\nabla_\mu N^\mu\rangle\,. \tag{3.15}$$

This is the consistency condition that we now study in detail.

3.3 Consistent anomalies

As we have pointed out, it is the consistent form of anomalies that are relevant to conservation laws of gauge currents. We now evaluate the consistency condition (3.15) using (2.9) for the anomalies. We specialize to the gauge group $G \times \mathrm{U}(1)$. As in section 2.2 we use T^a and iQ for the generators and A^a_μ and C_μ for the gauge fields, $F^a_{\mu\nu}$ and $C_{\mu\nu}$ for their field strengths. If necessary, we label abelian quantities by Q, but we drop the label a on the Fayet–Iliopoulos coupling ξ of the single $\mathrm{U}(1)$. This notation should not be confused with the previous section where a was an index of the full gauge group, not just G. In particular, we now write $F^a = 0$ and $F^Q = i\xi$ instead of (3.12). With these simplifications, the gauge potentials coupling in the left-chiral covariant derivatives (3.6) are

$$\begin{aligned}
\Psi_\mu:\ & V_\mu = -\frac{i}{2}K_\mu\,, \\
\lambda^a:\ & V_\mu^{ab} = -A^c_\mu f^{abc} - \frac{i}{2}K_\mu\delta^{ab}\,, \\
L\psi^\alpha:\ & V_\mu = A^a_\mu T^a + iQC_\mu + \frac{i}{2}K_\mu\,.
\end{aligned} \tag{3.16}$$

The consistent anomalies are obtained by inserting the relevant connection V_μ for each of the three types of fermions in (2.9) and collect results.

Non-abelian consistency condition:

The non-abelian consistent anomaly of the chiral fermion current now reads

$$\begin{aligned}
-\langle\nabla_\mu(\bar\psi^i T^a_{ij}\gamma^\mu L\psi^j)\rangle = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\,T^a\Big[& i\Big\{\partial_\mu A_\nu\partial_\rho A_\sigma + \frac{1}{2}\partial_\mu(A_\nu A_\rho A_\sigma)\Big\} \\
& -\Big\{\partial_\mu K_\nu\partial_\rho A_\sigma + \frac{1}{4}\partial_\mu(K_\nu A_\rho A_\sigma)\Big\} \\
& -2Q\Big\{\partial_\mu C_\nu\partial_\rho A_\sigma + \frac{1}{4}\partial_\mu(C_\nu A_\rho A_\sigma)\Big\}\Big].
\end{aligned} \tag{3.17}$$

We recognize in the first line the standard purely non-abelian anomaly and in the two other lines the mixed $G^2 - \mathrm{U}(1)$ plus the mixed G^2-Kähler anomalies. We also have

$$\frac{1}{2}\langle\nabla_\mu(\bar\lambda^b f^{abc}\gamma^\mu\lambda^c)\rangle = -\frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}C_2(G)\Big[\partial_\mu K_\nu\partial_\rho A^a_\sigma + \frac{1}{8}f^{abc}\partial_\mu(K_\nu A^b_\rho A^c_\sigma)\Big]\,. \tag{3.18}$$

Abelian consistency condition:

The abelian consistent anomaly of the chiral fermion current is

$$\begin{aligned}
-\langle\nabla_\mu(\bar\psi^i iQ\delta_{ij}\gamma^\mu L\psi^j)\rangle = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\Big[& Q^3\partial_\mu C_\nu\partial_\rho C_\sigma + \frac{1}{4}Q\partial_\mu K_\nu\partial_\rho K_\sigma + Q^2\partial_\mu K_\nu\partial_\rho C_\sigma \\
& -Q\Big(\partial_\mu A_\nu\partial_\rho A_\sigma + \frac{1}{2}\partial_\mu(A_\nu A_\rho A_\sigma)\Big)\Big]\,,
\end{aligned} \tag{3.19}$$

which contains the $\mathrm{U}(1)^3$, mixed $\mathrm{U}(1)-$Kähler and mixed $G^2 - \mathrm{U}(1)$ anomalies. The anomaly of the last term of (3.8) is

$$\frac{1}{2}\xi\langle\nabla_\mu N^\mu\rangle = -\frac{1}{24\pi^2}\frac{1}{2}\xi\epsilon^{\mu\nu\rho\sigma}\Big[-\mathrm{tr}(Q^2)\partial_\mu C_\nu\partial_\rho C_\sigma - \mathrm{tr}(Q)\partial_\mu C_\nu\partial_\rho K_\sigma + \frac{1}{4}\Big(n_\lambda + 3 - n_\psi\Big)\partial_\mu K_\nu\partial_\rho K_\sigma + \Big(C_2(G) - \sum_r C(r)\Big)\Big(\partial_\mu A^a_\nu\partial_\rho A^a_\sigma + \frac{1}{4}f^{abc}\partial_\mu(A^a_\nu A^b_\rho A^c_\sigma)\Big)\Big] . \tag{3.20}$$

In the $\partial K\partial K$ term, $n_\lambda = \dim(\mathcal{G})$ is the total number of gauginos, n_ψ is the number of chiral fermions, and 3 is the gravitino contribution.

3.4 Anomaly cancellation with local counter terms

As discussed in section 2.2, non-gauge invariant local counter terms can remove or restructure the anomalies. We now apply the result of section 2.2 to the anomaly conditions in the previous subsection.

To start with consider the non-abelian consistency condition

$$-\langle\nabla_\mu(\bar{\psi}^i T^a_{ij}\gamma^\mu L\psi^j)\rangle + \frac{1}{2}\langle\nabla_\mu(\bar{\lambda}^b f^{abc}\gamma^\mu\lambda^c)\rangle = 0\,, \tag{3.21}$$

with the two contributions given by (3.17) and (3.18) above. A counter term

$$\mathcal{L}_{\mathrm{CAA}} = \frac{1}{12\pi^2}\,\epsilon^{\mu\nu\rho\sigma}C_\mu\mathrm{tr}\Big[Q\Big(A_\nu\partial_\rho A_\sigma + \frac{3}{4}A_\nu A_\rho A_\sigma\Big)\Big] \tag{3.22}$$

will cancel the $G^2 - \mathrm{U}(1)$ mixed non-abelian anomaly and promote the abelian mixed anomaly to covariant form. The mixed G^2-Kähler anomaly is analogous, except for an overall factor $C_2(G) - \sum_r C(r)$. The correct counter term is

$$\mathcal{L}_{\mathrm{KAA}} = \frac{1}{24\pi^2}\Big(C_2(G) - \sum_r C(r)\Big)\epsilon^{\mu\nu\rho\sigma}K_\mu\Big(A^a_\nu\partial_\rho A^a_\sigma + \frac{3}{8}f^{abc}A^a_\nu A^b_\rho A^c_\sigma\Big)\,. \tag{3.23}$$

With these counter terms the non-abelian consistency condition becomes

$$\begin{aligned} 0 &= -\theta^a\langle\nabla_\mu(\bar{\psi}^i T^a_{ij}\gamma^\mu L\psi^j)\rangle + \frac{1}{2}\theta^a\langle\nabla_\mu(\bar{\lambda}^b f^{abc}\gamma^\mu\lambda^c)\rangle + \delta_\theta\mathcal{L}_{\mathrm{CAA}} + \delta_\theta\mathcal{L}_{\mathrm{KAA}} \\ &= \frac{i}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\theta^a\,\mathrm{tr}\,T^a\Big[\partial_\mu A_\nu\partial_\rho A_\sigma + \frac{1}{2}\partial_\mu(A_\nu A_\rho A_\sigma)\Big]\,. \end{aligned} \tag{3.24}$$

Mixed terms have been removed and we are left with the unavoidable non-abelian G^3 anomaly. Its cancellation imposes the condition $\mathrm{tr}[T^a\{T^b, T^c\}] = 0$ on the matter spectrum.

Let us now turn to the abelian consistency condition

$$-\langle\nabla_\mu(\bar{\psi}^i iQ\delta_{ij}\gamma^\mu L\psi^j)\rangle + \frac{1}{2}\xi\langle\nabla_\mu N^\mu\rangle \ = \ 0\ . \tag{3.25}$$

The abelian gauge variations read

$$\delta_\Lambda C_\mu = \partial_\mu\Lambda\,, \quad \delta_\Lambda K_\mu = \xi\partial_\mu\Lambda\,, \tag{3.26}$$

and the counter terms $\mathcal{L}_{\rm CAA}$ and $\mathcal{L}_{\rm KAA}$ restructure the consistent mixed anomalies into gauge invariant form.

The abelian consistency condition becomes

$$\begin{aligned} 0 &= -\Lambda\langle\nabla_\mu(\bar\psi^i iQ\delta_{ij}\gamma^\mu L\psi^j)\rangle + \frac{1}{2}\xi\Lambda\langle\nabla_\mu N^\mu\rangle + \delta_\Lambda\mathcal{L}_{\rm CAA} + \delta_\Lambda\mathcal{L}_{\rm KAA} \\ &= \frac{1}{96\pi^2}\epsilon^{\mu\nu\rho\sigma}\Lambda\Bigg[\mathrm{tr}\left[\left(Q+\frac{1}{2}\xi\right)Q^2\right]C_{\mu\nu}C_{\rho\sigma} + \mathrm{tr}\left[\left(Q+\frac{1}{2}\xi\right)Q\right]K_{\mu\nu}C_{\rho\sigma} \\ &\qquad + \frac{1}{4}\left(\mathrm{tr}\left[Q+\frac{1}{2}\xi\right] - \frac{1}{2}\xi(n_\lambda+3)\right)K_{\mu\nu}K_{\rho\sigma} \\ &\qquad -3\left(\mathrm{tr}\left[\left(Q+\frac{1}{2}\xi\right)T^aT^b\right] + \frac{1}{2}\xi C_2(G)\delta^{ab}\right)F^a_{\mu\nu}F^b_{\rho\sigma}\Bigg], \end{aligned} \tag{3.27}$$

with $K_{\mu\nu} = \partial_\mu K_\nu - \partial_\nu K_\mu$.

We must also consider the two counter terms

$$\mathcal{L}_{\rm CKK} = -\frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}C_\mu K_\nu\partial_\rho K_\sigma\,, \qquad \mathcal{L}_{\rm KCC} = -\frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}K_\mu C_\nu\partial_\rho C_\sigma\,, \tag{3.28}$$

which allow further cancellation of anomalies in the abelian consistency condition. Their gauge variations (after integration by parts) are

$$\begin{aligned} \delta_\Lambda\mathcal{L}_{\rm CKK} &= -\frac{1}{96\pi^2}\epsilon^{\mu\nu\rho\sigma}\Lambda(\xi C_{\mu\nu}K_{\rho\sigma} - K_{\mu\nu}K_{\rho\sigma})\,, \\ \delta_\Lambda\mathcal{L}_{\rm KCC} &= -\frac{1}{96\pi^2}\epsilon^{\mu\nu\rho\sigma}\Lambda(C_{\mu\nu}K_{\rho\sigma} - \xi C_{\mu\nu}C_{\rho\sigma})\,. \end{aligned} \tag{3.29}$$

We add $a_{\rm CKK}\mathcal{L}_{\rm CKK} + a_{\rm KCC}\mathcal{L}_{\rm KCC}$ to the Lagrangian and list below the independent terms from (3.27) and (3.24):

$$\begin{aligned} C\tilde{C}\ &:\ 0 = \mathrm{tr}\left[\left(Q+\frac{1}{2}\xi\right)Q^2\right] + \xi\, a_{\rm KCC}\,, \\ C\tilde{K}\ &:\ 0 = \mathrm{tr}\left[\left(Q+\frac{1}{2}\xi\right)Q\right] - \xi\, a_{\rm CKK} - a_{\rm KCC}\,, \\ K\tilde{K}\ &:\ 0 = \mathrm{tr}\, Q - \frac{1}{2}\xi(n_\lambda+3-n_\psi) + 4a_{\rm CKK}\,, \\ F^a\tilde{F}^b\ &:\ 0 = \mathrm{tr}\left[QT^aT^b\right] + \frac{1}{2}\xi\left[C_2(G) - \sum_r C(r)\right]\delta^{ab}\,, \\ G^3\ &:\ 0 = \mathrm{tr}\left[T^a\{T^b,T^c\}\right]\,. \end{aligned} \tag{3.30}$$

Each of the conditions (3.30) must be satisfied separately. Given the original field content of the model, these are the final and physical conditions for the cancellation of gauge anomalies. Gravitational anomalies will be considered next, and we will add new fields required by the Green–Schwarz mechanism in section 4.

There are additional consistency conditions from the gravitational anomalies with and external gauge current and two energy-momentum tensors in the triangle diagram. The resulting consistency condition is

$$\begin{aligned} 0 &= -\langle \nabla_\mu (\bar{\psi}^i iQ\delta_{ij}\gamma^\mu L\psi^j\rangle_{\rm grav} + \frac{1}{2}\xi\langle \nabla_\mu N^\nu\rangle_{\rm grav} \\ &= -\frac{1}{768\pi^2}\left[{\rm tr}(Q) - \frac{1}{2}\xi(n_\lambda - 21 - n_\psi)\right]\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\xi\tau}R_{\rho\sigma}{}^{\xi\tau}\ . \end{aligned} \tag{3.31}$$

For models with $\xi = 0$, one can choose $a_{\rm KCC}$ and $a_{\rm CKK}$ to satisfy the second and third conditions of (3.30). The remaining conditions reduce to the conventional anomaly cancellation conditions of a gauge theory coupled to gravity, namely the four traces ${\rm tr}\, Q$, ${\rm tr}\, Q^3$, ${\rm tr}\, QT^aT^b$, and ${\rm tr}\, T^a\{T^b, T^c\}$ must vanish. Note that without the counter term contribution (3.28), the second term in (3.30) would be positive definite and could never be cancelled by adjustment of the matter field content.

We now consider models with non-vanishing Fayet–Iliopoulos coupling. There are various cases of interest.

The first case is just an abelian vector multiplet coupled to supergravity and no chiral multiplets [1]. In this model $K_\mu = \xi C_\mu$; hence the counter terms (3.28) vanish. The only gauge anomaly condition which remains is $0 = \xi(n_\lambda + 3) = 4\xi$. The gravitational anomaly reduces to $0 = \xi(n_\lambda - 21) = -20\xi$. Clearly the model is inconsistent for $\xi \neq 0$.

In general models, we now show that it is very unlikely that the consistency conditions can be satisfy for non-vanishing ξ. First we choose the counter term coefficients $a_{\rm KCC}$ and $a_{\rm CKK}$ to satisfy the first two conditions of (3.30) and substitute the value of $a_{\rm CKK}$ into the third condition. We then replace $n_\lambda - n_\psi$ by the value determined by (3.31). The result is

$$0 = {\rm tr}\left[\Big(Q + \frac{1}{2}\xi\Big)\Big(Q + \xi\Big)Q\right] - 3\xi^3\,. \tag{3.32}$$

Consistency now requires that we satisfy the G^3 and $F\tilde{F}$ conditions of (3.30), and the conditions (3.32) and (3.31). A solution would require that both conditions linear in ξ have a common solution which is then one of the roots of the cubic condition (3.32). For given matter content this is extremely unlikely. This conclusion can be changed using a Green–Schwarz mechanism, as we discuss in the next section.

4. Green–Schwarz anomaly cancellation

The Green–Schwarz mechanism for anomaly cancellation in four dimensions is well known [5, 4]. One adds a chiral multiplet with a gauged shift symmetry. Decomposing the complex scalar s of the chiral multiplet as $s = \rho + ia$, the bosonic terms of the Green–Schwarz Lagrangian can then be written as

$$\mathcal{L}_{\rm GS} = -(\partial_\mu\rho)^2 - (\partial_\mu a + c_{\rm GS}C_\mu)^2 + \frac{1}{96\pi^2}a\,\epsilon^{\mu\nu\rho\sigma}\partial_\mu\Omega_{\nu\rho\sigma}\ . \tag{4.1}$$

where the Chern–Simons form $\Omega_{\nu\rho\sigma}$ satisfies

$$\epsilon^{\mu\nu\rho\sigma}\partial_\mu\Omega_{\nu\rho\sigma} = \epsilon^{\mu\nu\rho\sigma}\Big[b_{\rm CC}C_{\mu\nu}C_{\rho\sigma} + b_{\rm CK}K_{\mu\nu}C_{\rho\sigma} + b_{\rm KK}K_{\mu\nu}K_{\rho\sigma} + b_{\rm AA}F^a_{\mu\nu}F^a_{\rho\sigma} + b_{\rm RR}R_{\mu\nu\eta\tau}R_{\rho\sigma}{}^{\eta\tau}\Big]\ . \tag{4.2}$$

In a model originating from string theory [6, 25–28] (see also [29]), the constants $b_{..}$ will be fixed, but we keep them arbitrary here to illustrate their role in anomaly cancellation.

The scalar s is invariant under non-abelian gauge transformation, so $\delta_\theta\mathcal{L}_{\rm GS} = 0$. Under abelian gauge transformations,

$$\delta_\Lambda a = -c_{\rm GS}\Lambda\,, \quad \delta_\Lambda\rho = 0\ . \tag{4.3}$$

The first term in (4.1) is then gauge invariant, and it is then the last term whose gauge variation modifies the previous conditions (3.30) as follows:

$$\begin{aligned}
C\tilde{C}\ &:\ 0 = {\rm tr}\Big[\Big(Q+\frac{1}{2}\xi\Big)Q^2\Big] + \xi\, a_{\rm KCC} - c_{\rm GS}\, b_{\rm CC}\,,\\
C\tilde{K}\ &:\ 0 = {\rm tr}\Big[\Big(Q+\frac{1}{2}\xi\Big)Q\Big] - \xi\, a_{\rm CKK} - a_{\rm KCC} - c_{\rm GS}\, b_{\rm CK}\,,\\
K\tilde{K}\ &:\ 0 = {\rm tr}\, Q - \frac{1}{2}\xi(n_\lambda + 3 - n_\psi) + 4a_{\rm CKK} - 4c_{\rm GS}\, b_{\rm KK}\,,\\
F^a\tilde{F}^b\ &:\ 0 = {\rm tr}\Big[QT^aT^b\Big] + \frac{1}{2}\xi\Big[C_2(G) - \sum_r C(r)\Big]\delta^{ab} + \frac{1}{3}c_{\rm GS}\, b_{\rm AA}\delta^{ab}\,,\\
G^3\ &:\ 0 = {\rm tr}\Big[T^a\{T^b,T^c\}\Big]\,,\\
R\tilde{R}:\ &\ 0 = {\rm tr}(Q) - \frac{1}{2}\xi(n_\lambda - 21 - n_\psi) + 8c_{\rm GS}\, b_{\rm RR}\,.
\end{aligned} \tag{4.4}$$

Here n_ψ includes the contribution from the fermion partner χ of the Green–Schwarz scalar s. It is now evident that there is enough flexibility to cancel all gauge anomalies, and the only condition that needs to be imposed on the spectrum is ${\rm tr}[T^a\{T^b,T^c\}] = 0$ for the irreducible non-abelian anomaly. In fact, there is more flexibility than needed. We can set $b_{\rm CK} = b_{\rm KK} = 0$ and thus eliminate the composite connection completely from the Green–Schwarz Lagrangian. The remaining parameters then suffice to cancel all but the G^3 anomaly and allow an arbitrary value of the Fayet–Iliopoulos coupling.

The Green–Schwarz mechanism has served well to cancel the anomalies, but it has changed the physics of the model. To see this note that the a-C_μ cross term can be removed by an appropriate gauge fixing condition. This leaves a mass term

$$-c_{\rm GS}^2 C_\mu C^\mu\ . \tag{4.5}$$

Because of the gauged shift symmetry, the supersymmetric Lagrangian also contains a gauge invariant mass term $c_{\rm GS}\bar{\lambda}_C\chi$, where λ_C is gaugino partner of C_μ. This gives a fermion mass equal to that of the gauge bosons.

¿From the general form $D_\mu s = \partial_\mu s - X^{as}A^a_\mu$ we can formally identify the Killing vector

$$X^s = -ic_{\rm GS}\ . \tag{4.6}$$

The original scalars z^α and the Green–Schwarz scalar s combine in the gauge invariant Kähler potential

$$K(s,\bar{s},z,\bar{z}) = \delta_{\alpha\bar{\beta}} z^\alpha z^{\bar{\beta}} + \frac{1}{2}(s+\bar{s})^2 \,. \tag{4.7}$$

The new contribution from s to the Kähler connection is gauge invariant, so $\mathrm{Im}\, F^a = \xi$ is not changed, and the anomaly analysis is unmodified.

¿From (3.3) with $F^a = i\xi$ we find the U(1) D-term

$$D = iX^s K_{,s} + iX^\alpha K_{,\alpha} + \xi = \delta_{\alpha\bar{\beta}} z^\alpha Q z^{\bar{\beta}} + 2c_{\mathrm{GS}}\rho + \xi \,. \tag{4.8}$$

The scalar potential of supergravity contains the term $\frac{1}{2}D^2$. This term is minimized at $D = 0$, which can be achieved by adjusting ρ. Thus the breaking of the U(1) symmetry does not change the vacuum energy.

5. Generalizations

5.1 Non-gauge invariant Kähler potentials

It is interesting to examine the effect of a gauge non-invariant Kähler potential in the analysis of the gauge consistency conditions. We consider as in (3.11) the simplest model of flat target space $\mathbb{C}^n$ with linearly realized gauge symmetries. For simplicity, we exclude U(1) factors and gauge only a non-abelian simple subgroup of SU(n). However, contrary to the gauge invariant Kähler potential (3.11) we take here the unconventional Kähler potential

$$K(z,\bar{z}) = \delta_{\alpha\bar{\beta}} z^\alpha z^{\bar{\beta}} + k(z) + \bar{k}(\bar{z}) \,. \tag{5.1}$$

Here k is a non-constant holomorphic function, whose gauge variation (3.2) generates

$$F^a = X^{a\alpha} k_{,\alpha} \,. \tag{5.2}$$

It is somewhat artificial to break gauge symmetry by taking such an unnatural Kähler potential. However, most of our analysis — including the Green–Schwarz cancellation mechanism — applies to theories on non-flat target spaces in which gauge symmetry breaking in the Kähler potential cannot be avoided.

The gauge consistency conditions are again a special case of the result of [7]. Since $\mathrm{Im}\, F^a \neq 0$ the non-abelian consistency condition (3.21) includes the contribution from the divergence of the Noether current and now reads

$$\begin{aligned} 0 &= -\langle \nabla_\mu (\bar{\psi}^i T^a_{ij} \gamma^\mu L \psi^j) \rangle + \frac{1}{2} \langle \nabla_\mu (\bar{\lambda}^b f^{abc} \gamma^\mu \lambda^c) \rangle + \frac{1}{2} \mathrm{Im}\, F^a \langle \nabla_\mu N^\mu \rangle \\ &= \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \Bigg[\mathrm{tr}\, iT^a \Big\{ \partial_\mu A_\nu \partial_\rho A_\sigma + \frac{1}{2} \partial_\mu (A_\nu A_\rho A_\sigma) \Big\} \\ &\qquad - \widehat{\mathrm{tr}}\, T^a \Big\{ \partial_\mu K_\nu \partial_\rho A_\sigma + \frac{1}{4} \partial_\mu (K_\nu A_\rho A_\sigma) \Big\} \\ &\qquad - \frac{1}{2} \mathrm{Im}\, F^a \; \widehat{\mathrm{tr}} \Big\{ \partial_\mu A_\nu \partial_\rho A_\sigma + \frac{1}{2} \partial_\mu (A_\nu A_\rho A_\sigma) \Big\} \\ &\qquad - \frac{1}{8} \mathrm{Im}\, F^a \; (n_\lambda + 3 - n_\psi) \partial_\mu K_\nu \partial_\rho K_\sigma \Bigg] \,. \end{aligned} \tag{5.3}$$

JHEP11(2006)068

In the second equality we have inserted the expressions for the anomalies (3.17), (3.18), and (3.20) with the abelian ξ replaced by $\mathrm{Im}\,F^a$. We have also dropped the U(1) contributions by setting $Q = 0$. For brevity, we have introduced the notation $\widehat{\mathrm{tr}} = \mathrm{tr} - \mathrm{tr}_{\mathrm{adj}}$, i.e. the trace over chiral fermions minus the trace over gauginos. The minus sign in $\widehat{\mathrm{tr}}$ arises in (5.3) because the Kähler connection couples to chiral fermions and gauginos with opposite signs.

Before discussing the gauge consistency conditions (5.3) and the effect of local counter terms, we comment on Wess-Zumino (WZ) consistency (2.8). First recall that in the analysis of the previous sections, $\mathrm{Im}\,F^a$ vanished so that K_μ was invariant under non-abelian gauge transformations. The first two lines of (5.3) were then the only contributions to the gauge consistency condition and each of them independently satisfied the WZ consistency condition (2.8). In the present model, though, the Kähler connection does transform under non-abelian gauge transformations, $\delta_\theta K_\mu = \partial_\mu(\mathrm{Im}\,F^a\theta^a)$, and that gives an extra contribution to the variation of the anomaly in the second line of (5.3); by itself the second line of (5.3) is no longer WZ consistent.

It turns out that WZ consistency is saved by contributions from $\frac{1}{2}\mathrm{Im}\,F^a\langle\nabla_\mu N^\mu\rangle$. The $\partial K\partial K$ contribution in the last line of (5.3) satisfies WZ consistency because of the non-abelian transformation of $\mathrm{Im}\,F^a$. The gauge variation of the third line in (5.3) has two contributions: the variation of $\mathrm{Im}\,F^a$ yields the term required by WZ consistency, but the variation of $\widehat{\mathrm{tr}}\,[dAdA + (1/2)d(A^3)]$ gives an extra term. Conveniently, that extra term precisely cancels the unwanted term from the variation of K_μ in the second line of (5.3). Thus the full expression (5.3) does indeed satisfy the WZ consistency condition (2.8).

Returning to the analysis of the gauge consistency condition (5.3) we note that since K_μ transforms as an abelian connection under non-abelian gauge variations, the local counter term $\mathcal{L}_{\mathrm{KAA}}$ given in (3.23) removes the anomaly in the second line of (5.3) and it simultaneously converts the third line to covariant form. Including $\mathcal{L}_{\mathrm{KAA}}$ in the Lagrangian, the physically relevant form of the gauge consistency condition becomes

$$
\begin{aligned}
0 &= -\langle\nabla_\mu(\bar{\psi}^i T^a_{ij}\gamma^\mu L\psi^j)\rangle + \frac{1}{2}\langle\nabla_\mu(\bar{\lambda}^b f^{abc}\gamma^\mu\lambda^c)\rangle + \frac{1}{2}\mathrm{Im}\,F^a\langle\nabla_\mu N^\mu\rangle + \delta^a_\theta\mathcal{L}_{\mathrm{KAA}} \\
&= \epsilon^{\mu\nu\rho\sigma}\Bigg[\frac{1}{24\pi^2}\mathrm{tr}\,iT^a\Big\{\partial_\mu A_\nu\partial_\rho A_\sigma + \frac{1}{2}\partial_\mu(A_\nu A_\rho A_\sigma)\Big\} \\
&\qquad -\frac{1}{32\pi^2}\,\frac{1}{2}\mathrm{Im}\,F^a\ \widehat{\mathrm{tr}}\,F_{\mu\nu}F_{\rho\sigma} - \frac{1}{96\pi^2}\,\frac{1}{2}\mathrm{Im}\,F^a\ \frac{1}{4}(n_\lambda + 3 - n_\psi)K_{\mu\nu}K_{\rho\sigma}\Bigg].
\end{aligned}
\tag{5.4}
$$

For non-vanishing $\mathrm{Im}\,F^a$, consistency requires, besides the usual G^3 anomaly condition, that $C_2(G) = \sum_r C(r)$ and $n_\lambda + 3 - n_\psi = 0$. In addition, cancellation of the gravitational anomaly requires $n_\lambda - 21 - n_\psi = 0$. It is clear that these conditions cannot be simultaneously be satisfied, and so the models are inconsistent. An example is the (non-supersymmetric) gaugino model considered as a toy example in [7]. The model has no chiral multiplets and no gravitino, and the gauge anomaly $K\tilde{K}$ renders the model inconsistent.

In the previous section, we successfully applied the Green–Schwarz mechanism to remove anomalies. The fact that K_μ transforms as a U(1) connection under non-abelian gauge transformations, suggests that a Green–Schwarz mechanism can remove covariant anomalies proportional to $\operatorname{Im} F^a$ from the non-abelian conservation law.

A Green–Schwarz mechanism with a Chern–Simons term $a\,\epsilon^{\mu\nu\rho\sigma}\partial_\mu\Omega_{\nu\rho\sigma}$ can be used to cancel the mixed non-abelian anomalies in (5.4) provided that the axion transforms under non-abelian gauge transformations as $\delta_\theta a = -k_{\rm GS}\operatorname{Im} F^a\theta^a$ for some constant $k_{\rm GS}$, see [5]. Holomorphic behavior of the Green–Schwarz complex scalar $s = \rho + ia$ requires that it transforms non-trivially as

$$\delta_\theta s = -k_{\rm GS}\, F^a\theta^a\,. \tag{5.5}$$

The gauge invariant supersymmetric kinetic term for s is obtained from the superfield Kähler potential

$$\frac{1}{2}(S+\bar S + k_{\rm GS}\,K^{(0)})^2\,, \tag{5.6}$$

where S is the chiral superfield whose lowest component is s and $K^{(0)} = K^{(0)}(Z,\bar Z, V)$ is the standard Kähler potential for the chiral superfields Z, involving the real superfield $V = V^aT^a$ of the vector multiplet. The full Kähler potential is now

$$K(z,\bar z,s,\bar s) = K^{(0)}(z,\bar z) + \frac{1}{2}\Big(s+\bar s + k_{\rm GS}K^{(0)}(z,\bar z)\Big)^2\,. \tag{5.7}$$

We label the original Kähler potential and metric with superscripts $^{(0)}$, i.e. $G^{(0)}_{\alpha\bar\beta} = K^{(0)}_{,\alpha\bar\beta}$. It follows from (5.7) that the scalar kinetic terms are

$$\begin{aligned}
&-G_{\alpha\bar\beta}D_\mu z^\alpha D^\mu z^{\bar\beta} - G_{\alpha\bar s}D_\mu z^\alpha D^\mu \bar s - G_{s\bar\beta}D_\mu s D^\mu z^{\bar\beta} - G_{s\bar s}D_\mu s D^\mu \bar s\\
&\quad = -G^{(0)}_{\alpha\bar\beta}D_\mu z^\alpha D^\mu z^{\bar\beta} - k_{\rm GS}(s+\bar s + k_{\rm GS}K^{(0)})G^{(0)}_{\alpha\bar\beta}D_\mu z^\alpha D^\mu z^{\bar\beta}\\
&\qquad -(D_\mu s + k_{\rm GS}K^{(0)}_{,\alpha}\,D_\mu z^\alpha)(D^\mu\bar s + k_{\rm GS}K^{(0)}_{,\bar\beta}\,D^\mu z^{\bar\beta})\,,
\end{aligned} \tag{5.8}$$

where we have used that (5.5) implies $X^{as} = -k_{\rm GS}F^a$ for the holomorphic Killing vector, so that $D_\mu s = \partial_\mu s + k_{\rm GS}\,F^a\,A^a_\mu$. The first term in (5.8) is just the standard z-$\bar z$ kinetic term, and the two other terms come from the Green–Schwarz Lagrangian. Using the identity

$$\frac{1}{2}\partial_\mu K^{(0)} + iK^{(0)}_\mu = K^{(0)}_{,\alpha}\partial_\mu z^\alpha + A^a_\mu F^a - X^{a\alpha}K^{(0)}_{,\alpha}A^a_\mu\,, \tag{5.9}$$

where $K^{(0)}_\mu$ is the original Kähler connection, we rewrite the last term of (5.8). Then the Green–Schwarz Lagrangian for the chiral scalars takes the form

$$\begin{aligned}
\mathcal{L}_{\rm GS} &= -\Big(\partial_\mu a + k_{\rm GS}K^{(0)}_\mu\Big)^2 - \Big[\partial_\mu\Big(\rho + \frac{1}{2}k_{\rm GS}K^{(0)}\Big)\Big]^2\\
&\quad -k_{\rm GS}\Big(2\rho + k_{\rm GS}K^{(0)}\Big)G^{(0)}_{\alpha\bar\beta}D_\mu z^\alpha D^\mu z^{\bar\beta} + \frac{1}{96\pi^2}\,a\,\epsilon^{\mu\nu\rho\sigma}\partial_\mu\Omega_{\nu\rho\sigma}\,.
\end{aligned} \tag{5.10}$$

Note that the Green–Schwarz Lagrangian includes a correction to the z-$\bar z$ kinetic term.

The Green–Schwarz scalars and the new term in the Kähler potential (5.7) contribute to the Kähler connection, giving

$$K_\mu = K^{(0)}_\mu + \Big(2\rho + k_{\rm GS}K^{(0)}\Big)\Big(\partial_\mu a + k_{\rm GS}K^{(0)}_\mu\Big)\,. \tag{5.11}$$

Let the Chern–Simons form $\Omega_{\nu\rho\sigma}$ satisfy

$$\epsilon^{\mu\nu\rho\sigma}\partial_\mu\Omega_{\nu\rho\sigma} = \epsilon^{\mu\nu\rho\sigma}\Big[b_{\rm KK}K_{\mu\nu}K_{\rho\sigma} + b_{\rm AA}F^a_{\mu\nu}F^a_{\rho\sigma} + b_{\rm RR}R_{\mu\nu\eta\tau}R_{\rho\sigma}{}^{\eta\tau}\Big]\,, \tag{5.12}$$

with $K_{\mu\nu}$ the field strength of the corrected Kähler connection (5.11). Since the gauge invariant correction to the Kähler potential (5.7) does not change the value of $\operatorname{Im} F^a$, the constants $b_{..}$ in (5.12) can be chosen to cancel the mixed anomalies in (5.4) as well as the gravitational anomaly proportional to $\operatorname{Im} F^a$. This leaves only the usual G^3 anomaly. We conclude that the models considered in this section can be consistent only when the Green–Schwarz mechanism with the composite connection is included.

As for the standard Green–Schwarz mechanism, there are corrections to the D-term potential. We find

$$D^a = D^{(0)^a}\Big[1 + k_{\rm GS}(2\rho + k_{\rm GS}K^{(0)})\Big]\,, \tag{5.13}$$

where $D^{(0)^a}$ is the D-term before the corrections from the Green–Schwarz mechanism. Again the D-term conditions can be solved by adjusting ρ to make $D^a = 0$.

5.2 Supersymmetrization

So far our analysis has only included the bosonic terms of the anomalies. Since we started with a supersymmetric theory it is natural to consider supersymmetrized forms of local counter terms and the Green–Schwarz mechanism. Supersymmetric versions of the Green–Schwarz Lagrangian are known [5, 4], so here we focus on the counter terms.

The consistent anomaly is not gauge invariant, so we cannot work in Wess-Zumino gauge and must resort to superfields. The superfield version of the covariant anomaly is straightforward, $\mathcal{A}_{\rm cov} \propto \int d^4x\, d^2\theta {\rm tr}\, i\Lambda W^\alpha W_\alpha + h.c.$, where W^α denotes the non-abelian superfield vector field strength and Λ is a chiral superfield.[6] Supersymmetric expressions for the difference between the consistent and covariant anomaly for a simple gauge group are complicated [30 – 33]. Some simplication occurs for the mixed $\mathrm{U}(1) - G^2$ abelian anomaly which can be obtained from [30, 31] and written as

$$\begin{aligned}\mathcal{A}^Q_{\text{mixed con}} - \mathcal{A}^Q_{\text{mixed cov}} &= \frac{1}{64\pi^2}\int d^4x\, d^4\theta \int_0^1 dg\, \delta_\Lambda C\, {\rm tr}\Big[Q\, X_g(A)\Big]\,, \\ X_g(A) &= \Big([\mathcal{D}^\alpha A, W_\alpha(A)] + [\bar{D}_{\dot\alpha}A, \bar{W}^{\dot\alpha}(A)] + \{A, \mathcal{D}^\alpha W_\alpha(A)\}\Big)_{A\to gA}\,.\end{aligned} \tag{5.14}$$

As observed in (2.17), a desired property is that the abelian variation of the counter term restructures the mixed abelian consistent anomaly to covariant form. From (5.14) we can directly read off that the counter term

$$\mathcal{L}_{\rm Sct}(A, C) = -\frac{1}{64\pi^2}\int d^4\theta \int_0^1 dg\, C\, {\rm tr}\Big[Q\, X_g(A)\Big]\,. \tag{5.15}$$

[6] In this section, we focus on global supersymmetry. We use spinor indices $\alpha, \dot\alpha$ for the components of Weyl fermions. We use standard superspace conventions, for relevant details see [30, 31].

has exactly this property. Note that this fixes the counter term only up to terms that are invariant under abelian transformations. The non-abelian variation of the counter term (5.15) should cancel the supersymmetrized version of the mixed $\mathrm{U}(1) - G^2$ non-abelian anomaly $\mathcal{A}^a_{\text{mixed con}}$. Due to the complicated structure of the non-abelian variation $\delta_\Theta A$, we have only confirmed the cancellation of $\mathcal{A}^a_{\text{mixed con}}$ at leading order.[7] However, in component form, the supersymmetric counter term (5.15) correctly reproduces the bosonic counter term (2.14).

There is another approach to the mixed consistent anomaly involving the descent equations.[8] This leads to a different form of the mixed consistent anomalies, and suggests a counter term involving the non-abelian Chern–Simons three-form whose superfield expression can be found in [34, 5]. The resulting superfield counter term is similar to (5.15).

We have discussed some issues associated with a superfield formulation of counter terms with the desired properties. Addition study is needed to recast the consistency conditions and the full structure of the local counter terms in manifestly supersymmetric form.

6. Conclusions

We have clarified the consistency conditions that follow from the current conservation law (3.15) in supergravity [7] for flat sigma-model target space and linearly realized gauge symmetries. The analysis shows that anomalies arising from the non-invariance of the composite Kähler connection under gauge transformations complicate the anomaly cancellation conditions.

Starting from the consistent anomaly [22, 23] and including all finite local counter terms we reduce the consistency conditions to a set of physically relevant conditions. For vanishing Fayet–Iliopoulos couplings the conditions simplify to the standard anomaly cancellation conditions well-known from the Standard Model or the MSSM. However, a non-vanishing Fayet–Iliopoulos coupling ξ gives more involved consistency conditions. The usual G^3 condition $\mathrm{Tr}\, T^a\{T^b, T^c\} = 0$ must hold, but the Fayet–Iliopoulos coupling modifies the cancellation of the gravitational and the mixed $\mathrm{U}(1) - G^2$ anomalies. Some anomalies may be removed by including finite local counter terms in the action. A consistent model then requires:

$$\begin{aligned}
G^3 \;&:\; 0 = \mathrm{tr}\Big[T^a\{T^b, T^c\}\Big]\,, \\
F^a\tilde{F}^b \;&:\; 0 = \mathrm{tr}\Big[QT^aT^b\Big] + \frac{1}{2}\xi\Big[C_2(G) - \sum_r C(r)\Big]\delta^{ab}\,, \\
R\tilde{R} \;&:\; 0 = \mathrm{tr}(Q) - \frac{1}{2}\xi(n_\lambda - 21 - n_\psi)\,, \\
\text{abelian} \;&:\; 0 = \mathrm{tr}\left[\Big(Q + \frac{1}{2}\xi\Big)\Big(Q + \xi\Big)Q\right] - 3\xi^3\,.
\end{aligned} \tag{6.1}$$

[7] The form of the consistent anomaly presented in [32, 33] may be more useful for this purpose since the expressions there involve only e^A and the complications of the non-abelian gauge variations $\delta_\Theta A$ do not arise.

[8] We thank Massimo Bianchi and Emilian Dudas for drawing our attention to this point, see [25].

Solving the full set of conditions to find a consistent model looks unlikely, nonetheless it would be curious to see if there actually are consistent models. It is clear that the Fayet–Iliopoulos coupling in such a model cannot be treated as an arbitrary parameter.

At the cost of including extra degrees of freedom, the Green–Schwarz mechanism provides enough flexibility to cancel anomalies for arbitrary values of the Fayet–Iliopoulos couplings.

An immediate consequence of adding the Green–Schwarz Lagrangian is a mass term for the abelian gauge boson and a modification of the D-term potential from the Green–Schwarz scalar. In other words, in the presence of a Fayet–Iliopoulos coupling the abelian gauge boson always gains a mass irrespective of the vacuum structure. Furthermore, the D-term is corrected by the contribution from the Green–Schwarz scalar s. It then reads

$$\frac{1}{2}D^2 = \frac{1}{2}\Big(\sum_i q_i|\phi_i|^2 + \xi + c_{\mathrm{GS}}K_{,s}\Big)^2\,, \tag{6.2}$$

for linearly transforming matter fields ϕ_i with $\mathrm{U}(1)$ charges q_i. The c_{GS}-term enters through the Green–Schwarz Lagrangian. Formally, this correction is a one-loop effect, and there may be more corrections at the same order in perturbation theory that affect the scalar potential. This means there are effectively no field-independent Fayet–Iliopoulos couplings.[9]

As a generalization we also study consistency conditions for models with non-invariant Kähler potentials. Again the models are assumed to have flat target space and linearly realized gauge symmetries. We focus on models with simple gauge groups, leaving the inclusion of $\mathrm{U}(1)$-factors as a possible generalization. While local counter terms remove mixed Kähler$-G^2$ anomalies from the non-abelian consistency condition, the conditions for cancelling $K\tilde{K}$ and gravitational anomalies cannot simultaneously be satisfied, and so the models are inconsistent as they stand.

The Green–Schwarz mechanism based on the composite connection and the Kähler potential can remove this type of non-abelian anomalies. Since the Kähler connection transforms as an abelian connection under non-abelian gauge variations, it is possible to arrange a Green–Schwarz Chern–Simons term to cancel the anomalies in the non-abelian current conservation law. The simple models with non-invariant Kähler potentials can be consistent only when this Green–Schwarz mechanism is included to cancel the anomalies. As in (6.2), there are corrections to the D-term potential.

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[9] The possibility to "integrate out" the new scalar by setting it to a constant background value in the D-term seems only feasable if supersymmetry gets broken in the process, see [15].

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JHEP11(2006)068